Blacktown Boys' High School

2019

HSC Trial Examination

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided for this paper
- All diagrams are not drawn to scale
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 3 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 8 – 12)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Assessor: X. Chirgwin

Student Name: ______________________________

Teacher Name: ______________________________

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2019 Higher School Certificate Examination.

Office Use Only

Question	Mark
Q1	/1
Q2	/1
Q3	/1
Q4	/1
Q5	/1
Q6	/1
Q7	/1
Q8	/1
Q9	/1
Q10	/1
Q11 a)	/1
Q11 b)	/3
Q11 c)	/3
Q11 d)	/3
Q11 e)	/3
Q11 f)	/2
Q12 a)	/2
Q12 b)	/3
Q12 c)	/5
Q12 d)	/3
Q12 e)	/2
Q13 a)	/5
Q13 b)	/5
Q13 c)	/5
Q14 a)	/3
Q14 b)	/2
Q14 c)	/5
Q14 d)	/5
Total	**/70**

Section I

10 marks
Attempt Questions 1–10

Use the multiple choice answer sheet provided on page 13 for Questions 1–10.

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1 The acute angle between the lines $y = 1 + 5x$ and $y = 9x$ is θ.

What is the value of $\tan\theta$?

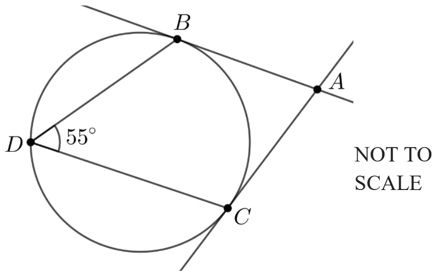
A. $\frac{5}{9}$

B. $\frac{7}{22}$

C. $\frac{2}{23}$

D. $\frac{1}{11}$

2 The diagram below shows a circle with tangents at AB and AC. D is a point on the circle such that $\angle BDC = 55°$.



Which of the following is true?

A. $\angle BAC = 70°$

B. $\angle BAC = 55°$

C. $\angle BAC = 50°$

D. $\angle BAC = 35°$

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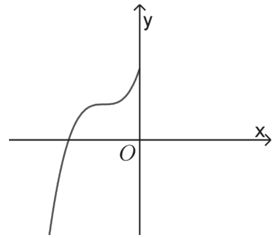
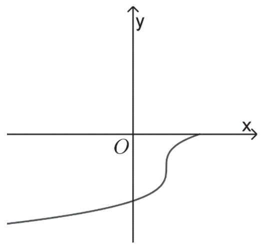
3 The diagram shows the graph $y = f(x)$.

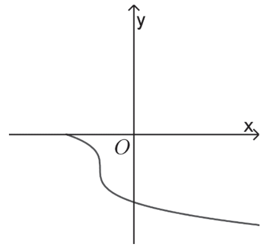


Which diagram shows the graph $y = f^{-1}(x)$?

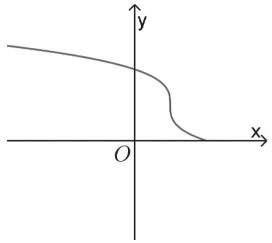
A.



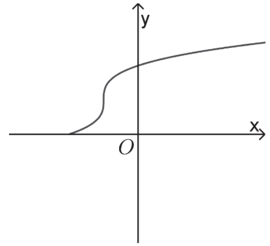
B.



C.



D.



4 What are the asymptotes of $y = \dfrac{10x}{(x-5)(x+2)}$?

A. $y = 0, x = 2, x = -5$

B. $y = 0, x = -2, x = 5$

C. $y = 10, x = 2, x = -5$

D. $y = 10, x = -2, x = 5$

5 When the polynomial $P(x)$ is divided by $(x-2)(x+3)$ the remainder is $(1-7x)$. What is the remainder when $P(x)$ is divided by $(x+3)$?

A. -20

B. -13

C. 15

D. 22

6 What is the general solution of the equation $2\cos^2 x - 9\cos x - 5 = 0$?

A. $x = 2n\pi \pm \frac{\pi}{6}$

B. $x = 2n\pi \pm \frac{5\pi}{6}$

C. $x = 2n\pi \pm \frac{2\pi}{3}$

D. $x = 2n\pi \pm \frac{\pi}{3}$

7 Cameron, Vishaal and five other friends sit randomly around a table. How many arrangements are possible if Cameron and Vishaal sit away from each other?

A. 120

B. 240

C. 480

D. 720

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8 A particle is moving in simple harmonic motion with period 6π and amplitude 5. Which of the following is a possible equation for the velocity of the particle?

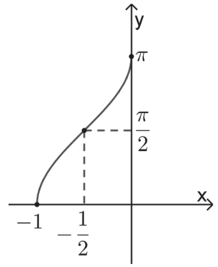
A. $v = 5\cos\frac{t}{6}$

B. $v = \frac{5}{3}\cos\frac{t}{6}$

C. $v = 5\cos\frac{t}{3}$

D. $v = \frac{5}{3}\cos\frac{t}{3}$

9 The diagram shows the graph of a function.



NOT TO SCALE

Which function does the graph represent?

A. $y = \frac{\pi}{2} - \sin^{-1}(2x-1)$

B. $y = \frac{\pi}{2} + \sin^{-1}(2x+1)$

C. $y = \cos^{-1}(2x+1)$

D. $y = \cos^{-1}(2x-1)$

10 Given that $\sum_{k=1}^{n} k^4 = \frac{8n^5 + an^4 + bn^3 - n}{38}$

The value of $a - b$ is:

A. 5

B. 4

C. 3

D. 2

End of Section I

Section II

60 Marks
Attempt Questions 11–14
Answer each question on a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Find $\int \frac{1}{x^2 + 8} dx$ **1**

b) Evaluate $\int_0^{\frac{\pi}{9}} \sin^2 3x \, dx$ **3**

c) Solve $\frac{3x}{2x + 5} \geq 1$ **3**

d) The letters A, E, I, O, and U are vowels.

i) How many arrangements of the letters in the word BINOMIAL are possible? **1**

ii) How many arrangements of the letters in the word BINOMIAL are possible if the vowels must occupy the first, third, fifth, and eighth positions? **2**

e) i) Show that a root of the continuous function $f(x) = \cos 4x - \ln x$ lies between $x = 1.2$ and $x = 1.3$. **1**

ii) Hence use one application of Newton's method with an initial estimate of $x = 1.2$ to find a second approximation to the zero. Write your answer correct to three significant figures. **2**

f) Find the exact value of $\tan\left(2\tan^{-1}\frac{\sqrt{3}}{5}\right)$ **2**

End of Questions 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

a) Find the constant term in the expansion of $\left(2x - \frac{3}{8x^3}\right)^{16}$. **2**

b) Use the substitution $u = x + 5$ to evaluate $\int_4^{11} \frac{x}{\sqrt{x+5}}\,dx$. **3**

c) A cup of hot chocolate, which is initially at a temperature of $75°C$, is placed on a table in the dining room to cool. The dining room has a constant temperature of $23°C$. The cooling rate of the cup of hot chocolate is proportional to the difference between the dining room temperature and the temperature, T, of the cup of hot chocolate. That is, T satisfies the equation

$$\frac{dT}{dt} = -k(T - 23),$$

where t is the number of minutes after the cup of hot chocolate is placed on the dining table.

i) Show that $T = 23 + Ae^{-kt}$ satisfies this equation, where A is a constant. **1**

ii) If the temperature of the cup of hot chocolate is $50°C$ after 10 minutes. Find the temperature of the cup of hot chocolate after 20 minutes. Round your answer to the nearest degree. **3**

iii) How long would it take for the cup of hot chocolate to cool to one third of its initial temperature? Round your answer to the nearest minute. **1**

d) The acceleration of a particle moving in a straight line is given by $\ddot{x} = -3e^{-2x}$, where x is the displacement from the origin. Initially the object is at the origin with velocity $v = \sqrt{3}\ ms^{-1}$.

i) Prove that $v = \sqrt{3}e^{-x}$. **2**

ii) What happens to v as x increases without bound? **1**

e) Show that $\lim_{x\to 0} \frac{1 - \cos 4x}{16x^2} = \frac{1}{2}$. **2**

End of Questions 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) A particle moves in a straight line and is x metres from a fixed point O after t seconds, where $x = 10 + \sqrt{3}\cos 2t - \sin 2t$.

i) Show that $\sqrt{3}\cos 2t - \sin 2t = 2\cos\left(2t + \frac{\pi}{6}\right)$. **1**

ii) Prove that the acceleration of the particle is $-4(x - 10)$. **2**

iii) Between which two points does the particle oscillate? **1**

iv) At what time does the particle first pass through the point $x = 10$? **1**

b) The point $P(2ap, ap^2)$ lies on the parabola $x^2 = 4ay$. The focus S is the point $(0, a)$. The tangent at P meets the y-axis at Q.

i) Derive the equation of the tangent at P and show that it is **2**

$$y = px - ap^2$$

ii) Find the co-ordinates of Q. **1**

iii) Show that the distance of SP is $a(p^2 + 1)$. **1**

iv) Hence prove that ΔQSP is an isosceles triangle. **1**

c) Consider the function $f(x) = 2\cos^{-1}\sqrt{x} - \cos^{-1}(2x - 1) + \pi$ for $0 \le x \le 1$.

i) Show that $f'(x) = 0$ for $0 < x < 1$. **3**

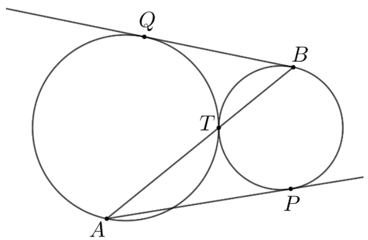
ii) Sketch the graph of $y = f(x)$. **2**

End of Questions 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) Use mathematical induction to prove that $9^{n+2} - 4^n$ is divisible by 5, for all positive integers n. **3**

b) The circles below touch at T. ATB is a straight line. AP is a tangent to circle PTB and BQ is a tangent to circle QTA. Prove that $AP^2 + BQ^2 = AB^2$. **2**



c) An object is projected from ground level at an angle θ to the horizontal, with a velocity of $V\ m/s$. The object returns to the ground after 50 seconds and 2 km from its point of projection.

i) Use $g = 10\ m/s^2$, show that the equations of the motion of this object is $x = Vt\cos\theta$ and $y = Vt\sin\theta - 5t^2$, where $t \leq 50$. **2**

ii) Hence find the exact value of V, and find angle θ to the nearest minute. **2**

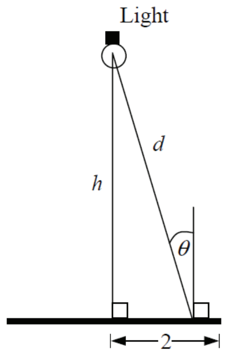
iii) What is the maximum height reached by the object? **1**

Question 14 continues on page 12

Question 14 (continued)

d) A light hangs at a vertical distance h metres above the centre of a circular table of radius 2 metres.

At any point on the table where the angle of incidence is θ and the distance from the light is d, as shown in the diagram, assume that the illumination I is given by $I = \dfrac{k\cos\theta}{d^2}$, where k is a positive constant.



i) Show that, at the edge of the table, $I = \dfrac{k\cos\theta\sin^2\theta}{4}$. **1**

ii) The vertical height of the light above table is varied. Given that $\cos\theta\sin^2\theta$ has a maximum value when $\theta = \cos^{-1}\dfrac{1}{\sqrt{3}}$, find the value of h that gives the maximum illumination at the edge of the table. **1**

iii) If the light is raised vertically at 0.16 ms^{-1}, find an expression for $\dfrac{d\theta}{dt}$. **1**

iv) Hence, or otherwise, find $\dfrac{dI}{dt}$ at the edge of the table when the light is 2 metres above the table. **2**

End of Paper

Student Name:____

Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word 'correct' and drawing an arrow as follows.

correct

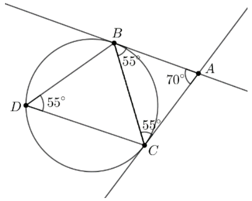
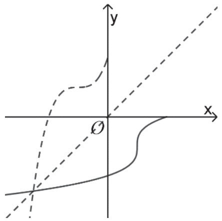
A ⊗ B ⊗ C ○ D ○

Start Here →

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

2019 Mathematics Extension 1 AT4 Trial Solutions

Section 1

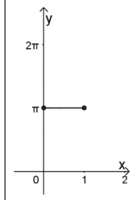
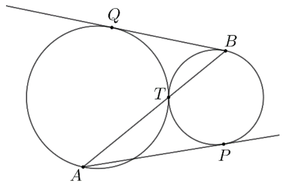
1	**C** Line 1: $y = 1 + 5x,\ m_1 = 5$ Line 2: $y = 9x,\ m_2 = 9$ $\tan\theta = \left\lvert\frac{9-5}{1+9\times 5}\right\rvert$ $\tan\theta = \frac{2}{23}$	1 Mark
2	**A**  Join BC $\angle ABC = \angle BDC = 55°$ (Alternate Segment Theorem) $\angle ACB = \angle BDC = 55°$ (Alternate Segment Theorem) $\angle ACB + \angle ABC + \angle BAC = 180°$ (Angle sum of ΔABC) $\angle BAC = 180° - 55° - 55°$ $\angle BAC = 70°$	1 Mark
3	**A**  	1 Mark
4	**B** Vertical asymptotes at $x = -2, x = 5$ Horizontal asymptote at $y = 0$ $y = \frac{10x}{(x-5)(x+2)}$ $y = \frac{10x}{x^2 - 3x - 10}$ $y = \frac{\frac{10x}{x^2}}{\frac{x^2}{x^2} - \frac{3x}{x^2} - \frac{10}{x^2}}$ $x \to \infty, y \to 0$	1 Mark
5	**D** $P(x) = (x-2)(x+3)Q(x) + (1-7x)$ $P(-3) = (-3-2)(-3+3)Q(3) + (1 - 7\times 3)$ $P(-3) = 22$ ∴ the remainder is 22	1 Mark

6	**C** $2\cos^2 x - 9\cos x - 5 = 0$ $(2\cos x + 1)(\cos x - 5) = 0$ $\cos x = -\frac{1}{2}$ $x = 2n\pi \pm \frac{2\pi}{3}$	1 Mark
7	**C** The arrangement of 7 people sitting around a table is 6! The arrangement of Cameron and Vishaal sit together is $5! \times 2!$ The arrangement of Cameron and Vishaal sit apart is the complement to Cameron and Vishaal sit together, which is $6! - 5! \times 2! = 480$	1 Mark
8	**D** $T = \frac{2\pi}{n}$ $T = \frac{2\pi}{6\pi}$ $T = \frac{1}{3}$ $x = 5\sin\frac{t}{3}$ $\therefore v = \frac{5}{3}\cos\frac{t}{3}$	1 Mark
9	**B** $y = \frac{\pi}{2} + \sin^{-1}(2x+1)$	1 Mark
10	**A** $\sum_{k=1}^{n} k^4 = \frac{8n^5 + an^4 + bn^3 - n}{38}$ $1^4 = \frac{8 \times 1^5 + a \times 1^4 + b \times 1^3 - 1}{38}$ $a + b = 31 \ldots\ldots\ldots (1)$ $1^4 + 2^4 = \frac{8 \times 2^5 + a \times 2^4 + b \times 2^3 - 2}{38}$ $17 = \frac{256 + 16a + 8b - 2}{38}$ $16a + 8b = 392$ $2a + b = 49 \ldots\ldots\ldots (2)$ $(2) - (1)$ $a = 18$ $b = 13$ $\therefore a - b = 5$	1 Mark

Section 2		
Q11 a)	$\int \frac{1}{x^2+8}dx$ $= \frac{1}{\sqrt{8}}\tan^{-1}\left(\frac{x}{\sqrt{8}}\right) + C$ $= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{2\sqrt{2}}\right) + C$ $= \frac{\sqrt{2}}{4}\tan^{-1}\left(\frac{\sqrt{2}x}{4}\right) + C$	1 Mark Correct solution
Q11 b)	$\int_0^{\frac{\pi}{9}} \sin^2 3x\, dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{9}}(1 - \cos 6x)dx$ $= \frac{1}{2}\left[x - \frac{1}{6}\sin 6x\right]_0^{\frac{\pi}{9}}$ $= \frac{1}{2}\left[\frac{\pi}{9} - \frac{1}{6}\sin\left(\frac{6\pi}{9}\right) - 0\right]$ $= \frac{1}{2}\left[\frac{\pi}{9} - \frac{1}{6} \times \frac{\sqrt{3}}{2}\right]$ $= \frac{\pi}{18} - \frac{\sqrt{3}}{24}$	3 Marks Correct solution 2 Marks Correct integral 1 Mark Find the correct relationship between $\cos 6x$ and $\sin^2 3x$
Q11 c)	$\frac{3x}{2x+5} \geq 1 \quad \left(x \neq -\frac{5}{2}\right)$ $3x(2x+5) \geq (2x+5)^2$ $3x(2x+5) - (2x+5)^2 \geq 0$ $(2x+5)[3x - (2x+5)] \geq 0$ $(2x+5)(x-5) \geq 0$ $x < -\frac{5}{2}, x \geq 5$	3 Marks Correct solution 2 Marks Identifies both important values 1 Mark Multiplies both sides by the square of the denominator, or equivalent
Q11 d) i)	BINOMIAL 8 letters to be arranged with 2 'I's are identical. The number of arrangements $= \frac{8!}{2!} = 20160$	1 Mark Correct solution
Q11 d) ii)	4 consonants can be arranged in $4!$ Ways. 4 vowels with 2 'I's can be arranged in $\frac{4!}{2!}$ ways Total number of arrangements $= 4! \times \frac{4!}{2!} = 288$	2 Marks Correct solution 1 Mark Identifies the number of arrangements for consonants or vowels
Q11 e) i)	$f(x) = \cos 4x - \ln x$ $f(1.2) = \cos(4 \times 1.2) - \ln 1.2$ $f(1.2) = -0.0948 \ldots$ $f(1.3) = \cos(4 \times 1.3) - \ln 1.3$ $f(1.3) = 0.206 \ldots$ Since $f(1.2) < 0$ and $f(1.3) > 0$, a root exists between 1.2 and 1.3 .	1 Mark Correct solution

Q11 e) ii)	$f'(x) = -4\sin 4x - \frac{1}{x}$ $x_1 = 1.2 - \frac{f(1.2)}{f'(1.2)}$ $x_1 = 1.2 - \frac{\cos(4\times 1.2) - \ln 1.2}{-4\sin(4\times 1.2) - \frac{1}{1.2}}$ $x_1 = 1.230 \ldots$ $x_1 = 1.23$ (3 significant figures)	2 Marks Correct solution 1 Mark Correct $f'(x)$
Q11 f)	Let $\theta = \tan^{-1}\frac{\sqrt{3}}{5}$ $\quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$ $\quad \tan\theta = \frac{\sqrt{3}}{5}$ $\tan\left(2\tan^{-1}\frac{\sqrt{3}}{5}\right)$ $= \tan 2\theta$ $= \frac{2\tan\theta}{1-\tan^2\theta}$ $= \frac{2\times\frac{\sqrt{3}}{5}}{1-\frac{3}{25}}$ $= \frac{5\sqrt{3}}{11}$	2 Marks Correct solution 1 Mark Uses double angle formula correctly
Q12 a)	$\left(2x - \frac{3}{8x^3}\right)^{16}$ $= \sum_{k=0}^{16}\binom{16}{k}(2x)^{16-k}\left(-\frac{3}{8x^3}\right)^k$ $= \sum_{k=0}^{16}\binom{16}{k}2^{16-k}\times x^{16-k}\times(-3)^k\times 8^{-k}\,x^{-3k}$ Constant term occurs when x^0 $x^{16-k}\times x^{-3k} = x^0$ $16-k-3k=0$ $16-4k=0$ $k=4$ Constant term is: $\binom{16}{4}2^{16-4}\times(-3)^4\times 8^{-4}$ $= 1820\times 2^{12}\times 3^4\times 2^{-12}$ $= 147420$	2 Marks Correct solution 1 Mark Finds the correct values of k
Q12 b)	Let $u = x+5 \quad du = dx$ $x = 11, u = 16$ $x = 4, u = 9$ $\int_4^{11}\frac{x}{\sqrt{x+5}}dx$ $= \int_9^{16}\frac{u-5}{\sqrt{u}}du$ $= \int_9^{16}\left(u^{\frac{1}{2}} - 5u^{-\frac{1}{2}}\right)du$ $= \left[\frac{2}{3}u^{\frac{3}{2}} - 10u^{\frac{1}{2}}\right]_9^{16}$ $= \left[\left(\frac{2}{3}\times 16^{\frac{3}{2}} - 10\times 16^{\frac{1}{2}}\right) - \left(\frac{2}{3}\times 9^{\frac{3}{2}} - 10\times 9^{\frac{1}{2}}\right)\right]$ $= \frac{44}{3}$	3 Marks Correct solution 2 Marks Finds the correct primitive function 1 Mark Transform the definite integral by applying the given substitution
Q12 c) i)	$T = 23 + Ae^{-kt} \quad \rightarrow \quad Ae^{-kt} = T - 23$ $\frac{dT}{dt} = -Ake^{-kt}$ $\frac{dT}{dt} = -kAe^{-kt}$ $\frac{dT}{dt} = -k(T-23)$	1 Marks Correct solution
Q12 c) ii)	At $t = 0, T = 75°C$ $75 = 23 + Ae^0$ $A = 75 - 23$ $A = 52°C$ At $t = 10, T = 50°C$ $50 = 23 + 52e^{-10k}$ $27 = 52e^{-10k}$ $e^{-10k} = \frac{27}{52}$ $-10k = \ln\left(\frac{27}{52}\right)$ $k = \frac{\ln\left(\frac{27}{52}\right)}{-10}$ $k = 0.06554\ldots$ At $t = 20, T = ?$ $T = 23 + 52e^{-20k}$ $T = 23 + 52e^{-20\times\frac{\ln\left(\frac{27}{52}\right)}{-10}}$ $T = 37.019\ldots$ The temperature of the cup of hot chocolate after 20 minutes is approximately $37°C$.	3 Marks Correct solution 2 Marks Finds the value of A and k correctly 1 Mark Finds the value of A

Q12 c) iii)	$T = \frac{1}{3} \times 75°C = 25°C$ $25 = 23 + 52e^{-t \times \frac{\ln\left(\frac{27}{52}\right)}{-10}}$ $2 = 52e^{t \times \frac{\ln\left(\frac{27}{52}\right)}{10}}$ $\ln\left(\frac{2}{52}\right) = t \times \frac{\ln\left(\frac{27}{52}\right)}{10}$ $t = \ln\left(\frac{2}{52}\right) \div \frac{\ln\left(\frac{27}{52}\right)}{10}$ $t = 49.711 \ldots$ It will take approximately 50 minutes to cool to one third of its initial temperature.	1 Mark Correct solution
Q12 d) i)	$\ddot{x} = -3e^{-2x}$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -3e^{-2x}$ $\frac{1}{2}v^2 = \int -3e^{-2x}dx$ $\frac{1}{2}v^2 = \frac{3}{2}e^{-2x} + C$ When $x = 0, v = \sqrt{3}$ $\frac{1}{2} \times (\sqrt{3})^2 = \frac{3}{2}e^0 + C$ $\frac{3}{2} = \frac{3}{2} + C$ $C = 0$ $\frac{1}{2}v^2 = \frac{3}{2}e^{-2x}$ $v^2 = 3e^{-2x}$ $v = \pm(3e^{-2x})^{\frac{1}{2}}$ Since $x = 0, v = \sqrt{3}$ $v = \sqrt{3}e^{-x}$	2 Marks Correct solution 1 Mark Correct primitive function
Q12 d) ii)	$v = \sqrt{3}e^{-x}$ $v = \frac{\sqrt{3}}{e^x}$ $x \to \infty$ $e^x \to \infty$ $v \to 0$	1 Mark Correct solution
Q12 e)	$\lim_{x\to 0} \frac{1-\cos 4x}{16x^2}$ $= \lim_{x\to 0} \frac{1-(1-2\sin^2 2x)}{16x^2}$ $= \lim_{x\to 0} \frac{2\sin^2 2x}{16x^2}$ $= \lim_{x\to 0} \frac{\sin^2 2x}{8x^2}$ $= \frac{1}{2}\lim_{x\to 0} \frac{\sin^2 2x}{4x^2}$ $= \frac{1}{2}\lim_{x\to 0} \frac{(\sin 2x)^2}{(2x)^2}$ $= \frac{1}{2} \times 1^2$ $= \frac{1}{2}$	2 Marks Correct solution 1 Mark Correctly uses double angle result
Q13 a) i)	$2\cos\left(2t + \frac{\pi}{6}\right)$ $= 2\cos 2t \cos\frac{\pi}{6} - 2\sin 2t \sin\frac{\pi}{6}$ $= 2\cos 2t \times \frac{\sqrt{3}}{2} - 2\sin 2t \times \frac{1}{2}$ $= \sqrt{3}\cos 2t - \sin 2t$ $\therefore \sqrt{3}\cos 2t - \sin 2t = 2\cos\left(2t + \frac{\pi}{6}\right)$	1 Mark Correct solution
Q13 a) ii)	$x = 10 + \sqrt{3}\cos 2t - \sin 2t$ $\dot{x} = -2\sqrt{3}\sin 2t - 2\cos 2t$ $\ddot{x} = -4\sqrt{3}\cos 2t + 4\sin 2t$ $\ddot{x} = -4(\sqrt{3}\cos 2t - \sin 2t)$ $\ddot{x} = -4(x - 10)$	2 Marks Correct solution 1 Mark Differentiates to find correct $\ddot{x}$ in terms of t
Q13 a) iii)	$x = 10 + \sqrt{3}\cos 2t - \sin 2t$ $x = 10 + 2\cos\left(2t + \frac{\pi}{6}\right)$ $\therefore$ The particle is in simple harmonic motion, about the position 10, with amplitude 2. The particle oscillates between $x = 8$ and $x = 12$.	1 Mark Correct solution
Q13 a) iv)	$x = 10$ $10 + 2\cos\left(2t + \frac{\pi}{6}\right) = 10$ $2\cos\left(2t + \frac{\pi}{6}\right) = 0$ $2t + \frac{\pi}{6} = \frac{\pi}{2}$ $2t = \frac{\pi}{3}$ $t = \frac{\pi}{6}$ $\therefore$The particle first passes through $x = 10$ at time $t = \frac{\pi}{6}$ seconds.	1 Mark Correct solution

Q13 b) i)	$x^2 = 4ay$ $y = \frac{x^2}{4a}$ $\frac{dy}{dx} = \frac{2x}{4a}$ $\frac{dy}{dx} = \frac{x}{2a}$ At $P(2ap, ap^2)$ $m_T = \frac{2ap}{2a}$ $m_T = p$ Equation of tangent at P $y - ap^2 = p(x - 2ap)$ $y - ap^2 = px - 2ap^2$ $y = px - ap^2$	2 Marks Correct solution 1 Mark Find the gradient of tangent at P
Q13 b) ii)	At $Q, x = 0$ $y = -ap^2$ $Q(0, -ap^2)$	1 Mark Correct solution
Q13 b) iii)	$SP = \sqrt{(2ap - 0)^2 + (ap^2 - a)^2}$ $SP = \sqrt{4a^2p^2 + a^2p^4 - 2a^2p^2 + a^2}$ $SP = \sqrt{a^2p^4 + 2a^2p^2 + a^2}$ $SP = \sqrt{a^2(p^4 + 2p^2 + 1)}$ $SP = \sqrt{a^2(p^2 + 1)^2}$ $SP = a(p^2 + 1)$	1 Mark Correct solution
Q13 b) iv)	$SQ = a - (-ap^2)$ $SQ = a + ap^2$ $SQ = a(p^2 + 1)$ $SP = SQ$ $\therefore \Delta QSP$ is an isosceles triangle.	1 Mark Correct solution
Q13 c) i)	$f(x) = 2\cos^{-1}\sqrt{x} - \cos^{-1}(2x - 1)$ for $0 \le x \le 1$ $f'(x) = 2 \times -\frac{1}{\sqrt{1 - (\sqrt{x})^2}} \times \frac{1}{2}x^{-\frac{1}{2}} - \left(-\frac{1}{\sqrt{1 - (2x - 1)^2}} \times 2\right)$ $f'(x) = -\frac{1}{\sqrt{1 - x}} \times \frac{1}{\sqrt{x}} + \frac{2}{\sqrt{1 - (4x^2 - 4x + 1)}}$ $f'(x) = -\frac{1}{\sqrt{x - x^2}} + \frac{2}{\sqrt{4x - 4x^2}}$ $f'(x) = -\frac{1}{\sqrt{x - x^2}} + \frac{2}{2\sqrt{x - x^2}}$ $f'(x) = -\frac{1}{\sqrt{x - x^2}} + \frac{1}{\sqrt{x - x^2}}$ $f'(x) = 0$ $x - x^2 \ne 0$ otherwise $f'(x)$ is not defined $x(1 - x) \ne 0$ $x \ne 0, x \ne 1$ $f'(x) = 0$ for $0 < x < 1$	3 Marks Correct solution 2 Marks Differentiate correctly to find $f'(x) = 0$ 1 Mark Differentiate some parts of $f(x)$ correctly
Q13 c) ii)	$f(x)$ is a horizontal line for $0 \le x \le 1$ since $f'(x) = 0$, derivatives are not defined at $x = 0$ and $x = 1$ because they are endpoints of $f(x)$. $f(0) = 2\cos^{-1}\sqrt{0} - \cos^{-1}(2 \times 0 - 1) + \pi$ $f(0) = \pi$ $f(1) = 2\cos^{-1}\sqrt{1} - \cos^{-1}(2 \times 1 - 1) + \pi$ $f(1) = \pi$ 	2 Marks Correct solution 1 Mark Finds $f(x) = \pi$
Q14 a)	1. Prove statement is true for $n = 1$ $9^{1+2} - 4^1$ $= 725$ $= 5 \times 145$ $\therefore$ Statement is true for $n = 1$ 2. Assume statement is true for $n = k$ (k some positive integer) $9^{k+2} - 4^k = 5M$ (M some integer) 3. Prove statement is true for $n = k + 1$ $9^{k+1+2} - 4^{k+1} = 5Q$ (Q some integer) $LHS = 9^{k+3} - 4^{k+1}$ $LHS = 9 \times (9^{k+2} - 4^k) + 9 \times 4^k - 4 \times 4^k$ $LHS = 9 \times 5M + 9 \times 4^k - 4 \times 4^k$ (from step 2) $LHS = 9 \times 5M + (9 - 4) \times 4^k$ $LHS = 9 \times 5M + 5 \times 4^k$ $LHS = 5(9M + 4^k)$ $LHS = 5Q$ where $Q = 9M + 4^k$ $\therefore$ Statement is true by mathematical induction for all positive integer n.	3 Marks Correct solution 2 Marks Makes significant progress and uses assumed statement in the body of proof 1 Mark Proves statement is true for $n = 1$
Q14 b)	 $AP^2 = AB \times AT$ (square of tangent equals product of secant segments) $BQ^2 = AB \times BT$ (square of tangent equals product of secant segments) $AP^2 + BQ^2 = AB \times AT + AB \times BT$ $AP^2 + BQ^2 = AB \times (AT + BT)$ $AP^2 + BQ^2 = AB \times AB$ $AP^2 + BQ^2 = AB^2$	2 Marks Correct solution 1 Mark Recognises the relationship between tangent and secant

Q14 c) i)	Horizontal $\ddot{x} = 0$ $\dot{x} = C_1$ When $t = 0, \dot{x} = V\cos\theta, C_1 = V\cos\theta$ $\therefore \dot{x} = V\cos\theta$ $x = Vt\cos\theta + C_2$ When $t = 0, x = 0, C_2 = 0$ $\therefore x = Vt\cos\theta$ Vertical $\ddot{y} = -10$ $\dot{y} = -10t + C_3$ When $t = 0, \dot{y} = V\sin\theta, C_3 = V\sin\theta$ $\therefore \dot{y} = V\sin\theta - 10t$ $y = Vt\sin\theta - 5t^2 + C_4$ When $t = 0, y = 0, C_4 = 0$ $\therefore y = Vt\sin\theta - 5t^2$	2 Marks Correct solution 1 Mark Derive the equation of the motion for either x or y
Q14 c) ii)	When $t = 50, y = 0, x = 2000\ m$ $2000 = 50V\cos\theta$ $V\cos\theta = 40 \quad (1)$ $0 = 50V\sin\theta - 5\times 50^2$ $V\sin\theta = 250 \quad (2)$ $(2)\div(1)$ $\frac{V\sin\theta}{V\cos\theta} = \frac{250}{40}$ $\tan\theta = \frac{25}{4}$ $\theta = 80°54'35''$ $\theta = 80°55'$ (nearest minute) $(1)^2 + (2)^2$ $V^2\cos^2\theta + V^2\sin^2\theta = 40^2 + 250^2$ $V^2(\cos^2\theta + \sin^2\theta) = 64100$ $V^2 = 64100$ $V = 10\sqrt{641}\ m/s$	2 Marks Correct solution 1 Mark Finds the correct value for V or θ
Q14 c) iii)	$\tan\theta = \frac{25}{4}$ $\sin\theta = \frac{25}{\sqrt{641}}$ Maximum height when $\dot{y} = 0$ $V\sin\theta - 10t = 0$ $10\sqrt{641}\times\frac{25}{\sqrt{641}} - 10\times t = 0$ $t = 25$ $y = Vt\sin\theta - 5t^2$ $y = 10\sqrt{641}\times 25\times\frac{25}{\sqrt{641}} - 5\times 25^2$ $y = 3125\ m$ $\therefore$ Maximum height is $3125\ m$.	1 Mark Correct solution

Q14 d) i)	At the edge of the table $\sin\theta = \frac{2}{d}$ $d = \frac{2}{\sin\theta}$ $d^2 = \frac{4}{\sin^2\theta}$ $I = \frac{k\cos\theta}{d^2}$ $I = \frac{k\cos\theta}{\frac{4}{\sin^2\theta}}$ $I = k\cos\theta\times\frac{\sin^2\theta}{4}$ $\therefore I = \frac{k\cos\theta\sin^2\theta}{4}$	1 Mark Correct solution
Q14 d) ii)	$\theta = \cos^{-1}\frac{1}{\sqrt{3}}$ $\cos\theta = \frac{1}{\sqrt{3}}$ $\tan\theta = \frac{\sqrt{2}}{1}$ $\tan\theta = \frac{2}{h}$ $h = \frac{2}{\tan\theta}$ $h = 2\times\frac{1}{\sqrt{2}}$ $h = \sqrt{2}$ $\therefore h = \sqrt{2}\ m$ gives the maximum illumination at the edge of the table.	1 Mark Correct solution
Q14 d) iii)	$\frac{dh}{dt} = 0.16\ m/s \quad \frac{d\theta}{dt} = ?$ $h = \frac{2}{\tan\theta}$ $\frac{dh}{d\theta} = 2\times -1\times\sec^2\theta\times(\tan\theta)^{-2}$ $\frac{dh}{d\theta} = -2\times\frac{1}{\cos^2\theta}\times\frac{\cos^2\theta}{\sin^2\theta}$ $\frac{dh}{d\theta} = \frac{-2}{\sin^2\theta}$ $\frac{d\theta}{dt} = \frac{d\theta}{dh}\times\frac{dh}{dt}$ $\frac{d\theta}{dt} = \frac{\sin^2\theta}{-2}\times 0.16$ $\frac{d\theta}{dt} = -\frac{2\sin^2\theta}{25}$	1 Mark Correct solution

Q14 d) iv)	$I = \frac{k\cos\theta\sin^2\theta}{4}$ $\frac{dI}{d\theta} = \frac{k}{4}(-\sin\theta \times \sin^2\theta + \cos\theta \times 2\cos\theta\sin\theta)$ $\frac{dI}{d\theta} = \frac{k\sin\theta}{4}(-\sin^2\theta + 2\cos^2\theta)$ $\frac{dI}{d\theta} = \frac{k\sin\theta}{4}(\cos^2\theta - 1 + 2\cos^2\theta)$ $\frac{dI}{d\theta} = \frac{k\sin\theta}{4}(3\cos^2\theta - 1)$ $\frac{dI}{dt} = \frac{dI}{d\theta} \times \frac{d\theta}{dt}$ $\frac{dI}{dt} = \frac{k\sin\theta}{4}(3\cos^2\theta - 1) \times -\frac{2\sin^2\theta}{25}$ $h = 2\ m$ $\tan\theta = \frac{2}{2}$ $\tan\theta = 1$ $\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$ $\frac{dI}{dt} = \frac{k}{4} \times \frac{1}{\sqrt{2}} \times \left(3 \times \frac{1}{2} - 1\right) \times -\frac{2}{25} \times \frac{1}{2}$ $\frac{dI}{dt} = -\frac{k}{200\sqrt{2}}$ $\frac{dI}{dt} = -\frac{\sqrt{2}k}{400}\ m/s$	2 Marks Correct solution 1 Mark Find the correct expression of $\frac{dI}{d\theta}$